

Entanglement, intractability and no-signaling

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We consider the problem of deriving the no-signaling condition from the assumption that, as seen from a complexity theoretic perspective, the universe is not an exponential place. A fact that disallows such a derivation is the existence of *polynomial superluminal* gates, hypothetical primitive operations that enable superluminal signaling but not the efficient solution of intractable problems. It therefore follows, if this assumption is a basic principle of physics, either that it must be supplemented with additional assumptions to prohibit such gates, or, improbably, that no-signaling is not a universal condition. Yet, a gate of this kind is possibly implicit, though not recognized as such, in a decade-old quantum optical experiment involving position-momentum entangled photons. Here we describe a feasible modified version of the experiment that appears to explicitly demonstrate the action of this gate. Some obvious counter-claims are shown to be invalid. We believe that the unexpected possibility of polynomial superluminal operations arises because some practically measured quantum optical quantities are not describable as standard quantum mechanical observables.

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I. INTRODUCTION

In a multipartite quantum system, any completely positive (CP) map applied locally to one part does not affect the reduced density operator of the remaining part. This fundamental no-go result, called the “no-signalling theorem” implies that quantum entanglement [1] does not enable nonlocal (“superluminal”) signaling [2] under standard operations, and is thus consistent with relativity, inspite of the counterintuitive, stronger-than-classical correlations [3] that entanglement enables. For simple systems, no-signaling follows from non-contextuality, the property that the probability assigned to projector Π_x , given by the Born rule, $\text{Tr}(\rho\Pi_x)$, where ρ is the density operator, does not depend on how the orthonormal basis set is completed [4, 5]. No-signaling has also been treated as a basic postulate to derive quantum theory [6].

It is of interest to consider the question of whether/how computation theory, in particular intractability and uncomputability, matter to the foundations of (quantum) physics. Such a study, if successful, could potentially allow us to reduce the laws of physics to mathematical theorems about algorithms and thus shed new light on certain conceptual issues. For example, it could explain why stronger-than-quantum correlations that are compatible with no-signaling [7] are disallowed in quantum mechanics. One strand of thought leading to the present work, earlier considered by us in Ref. [8], was the proposition that the measurement problem is a consequence of basic algorithmic limitations imposed on the computational power that can be supported by physical laws. In the present work, we would like to see whether no-signaling can also be explained in a similar way, starting from computation theoretic assumptions.

The central problem in computer science is the conjecture that two computational complexity classes, **P** and **NP**, are distinct in the standard Turing model of computation. **P** is the class of decision problems solvable in polynomial time by a (deterministic) TM. **NP** is the class of decision problems whose solution(s) can be verified in polynomial time by a deterministic TM. **#P** is the class of counting problems associated with the decision problems in **NP**. The word “complete” following a class denotes a problem X within the class, which is maximally hard in the sense that any other problem in the class can be solved in poly-time using an oracle giving the solutions of X in a single clock cycle. For example, determining whether a Boolean formula is satisfied is **NP**-complete, and counting the number of Boolean satisfactions is **#P**-complete. The word “hard” following a class denotes a problem not necessarily in the class, but to which all problems in the class reduce in poly-time.

P is often taken to be the class of computational problems which are “efficiently solvable” (i.e., solvable in polynomial time) or “tractable”, although there are potentially larger classes that are considered tractable such as **RP** [9] and **BQP**, the latter being the class of decision problems efficiently solvable by a quantum computer [9]. **NP**-complete

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and potentially harder problems, which are not known to be efficiently solvable, are considered intractable in the Turing model. If $\mathbf{P} \neq \mathbf{NP}$ and the universe is a polynomial– rather than an exponential– place, physical laws cannot be harnessed to efficiently solve intractable problems, and \mathbf{NP} -complete problems will be intractable in the physical world.

That classical physics supports various implementations of the Turing machine is well known. More generally, we expect that computational models supported by a physical theory will be limited by that theory. Witten identified expectation values in a topological quantum field theory with values of the Jones polynomial that are $\#\mathbf{P}$ -hard [11]. There is evidence that a physical system with a non-Abelian topological term in its Lagrangian may have observables that are \mathbf{NP} -hard, or even $\#\mathbf{P}$ -hard [12].

Other recent related works that have studied the computational power of variants of standard physical theories from a complexity or computability perspective are, respectively, Refs. [8, 13–16] and Refs. [8, 13]. Ref. [15] noted that \mathbf{NP} -complete problems do not seem to be tractable using resources of the physical universe, and suggested that this might embody a fundamental principle, christened the \mathbf{NP} -hardness assumption (also cf. [17]). Ref. [18] studies how insights from quantum information theory could be used to constrain physical laws. We will informally refer to the proposition that the universe is a polynomial place in the computational sense (to be strengthened below) as well as the communication sense by the expression “the world is not hard enough” (WNHE) [19].

Recently, Ref. [20] have posed the question whether nonlinear quantum evolution can be considered as providing any help in solving otherwise hard problems, on the grounds that under nonlinear evolution, the output of such a computer on a mixture of inputs is not a convex combination of its output on the pure components of the mixture. We circumvent this problem here by adopting the standpoint of *information realism*, the position that physical states are ultimately information states registered in some way sub-physically but objectively by Nature. At this stage, we will not worry about the details except to note an implication for the present situation, which is that from the perspective of ‘Nature’s eye’, there are no mixed states. Therefore, in describing nonlinear physical laws or specifying the working of non-standard computers based on such laws, it suffices for our purpose to specify their action on (all relevant) pure state inputs.

In Ref. [8], we pointed out that the assumption of WNHE (and further that of $\mathbf{P} \neq \mathbf{NP}$) can potentially give a unified explanation of (a) the observed ‘insularity-in-theoryspace’ of quantum mechanics (QM), namely that QM is *exactly* unitary and linear, and requires measurements to conform to the $|\psi|^2$ Born rule [14, 21]; (b) the classicality of the macroscopic world; (c) the lack of quantum physical mechanisms for non-signaling superquantum correlations [7].

In (a), the basic idea is that departure from one or more of these standard features of QM seems to invest quantum computers with super-Turing power to solve hard problems efficiently, thus making the universe an exponential place, contrary to assumption. The possibility (b) arises for the following reason. It is proposed that the WNHE assumption holds not only in the sense that hard problems (in the standard Turing model) are not efficiently solvable in the physical world, but in the stronger sense that any physical computation can be simulated on a probabilistic TM with at most a polynomial slowdown in the number of steps (the Strong Church-Turing thesis). Therefore, the evolution of any quantum system computing a decision problem, could asymptotically be simulated in polynomial time in the size of the problem, and thus lies in \mathbf{BPP} , the class of problems that can be efficiently solved by a probabilistic TM [22].

Assuming $\mathbf{BPP} \neq \mathbf{BQP}$, this suggests that although at small scales, standard QM remains valid with characteristic \mathbf{BQP} -like behavior, at sufficiently large scales, classical (‘ \mathbf{BPP} -like’) behavior should emerge, and that therefore there must be a definite scale– sometimes called the Heisenberg cut– where the superposition principle breaks down [23], so that asymptotically, quantum states are not exponentially long vectors. In Ref. [8], we speculate that this scale is related to a discretization of Hilbert space. This approach provides a possible computation theoretic resolution to the quantum measurement problem. In (c), the idea is that in a polynomial universe, we expect that phenomena in which a polynomial amount of physical bits can simulate exponentially large (classical) correlations, thereby making communication complexity trivial, would be forbidden.

In the present work, we are interested in studying whether the no-signaling theorem follows from the WNHE assumption. The article is divided into two parts: Part I, concerned with the computer scientific aspects, giving a complexity theoretic motivation for the work; Part II, concerned with the quantum optical implementation of a test suggested by Part I.

In Part I, first some results concerning non-standard operations that violate no-signaling and help efficiently solve intractable problems are surveyed, in Sections IIA and IIB, respectively. Then, in Section IIC, we introduce the concept of a polynomial superluminal gate, a hypothetical primitive operation that is prohibited by the assumption of no-signaling, but allowed if instead we only assume that intractable problems should not be efficiently solvable by physical computers. We examine the relation between the above two classes of non-standard gates. We also describe a *constant* gate on a single qubit or qutrit, possibly the simplest instance of a polynomial superluminal operation.

In Part II, first we present a quantum optical realization of the constant gate, and its application to an experiment involving entangled light generated by parametric downconversion in a nonlinear crystal in Section III A. Physicists

who could not care less about computational complexity aspects could skip directly to this Section. They may be warned that the intervening sections of Part I will involve mangling QM in ways that may seem awkward, and whose consistency is, unfortunately, not obvious! On the other hand, computer scientists unfamiliar with quantum optics may skip Section III A, which is essentially covered in Section III B, which discusses quantitative and conceptual issues surrounding the physical realization of the constant gate. Finally, we conclude with Section IV by surveying some implications of a possible positive outcome of the proposed experiment, and discussing how such an unexpected physical effect may fit in with the mathematical structure of known physics. We present a slightly abridged version of discussions in this work in Ref. [24].

II. PART I: COMPUTATION THEORETIC MOTIVATION

A. Superluminal gates

Even minor variants of QM are known to lead to superluminal signaling. An example is a variant incorporating nonlinear observables [25], unless the nonlinearity is confined to sufficiently small scales [26–29]. In this Section, we will review the case of violation of no-signaling due to departure from standard QM via the introduction of (a) non-complete Schrödinger evolution or measurement, (b) nonlinear evolution [30], (c) departure from the Born $|\psi|^2$ rule.

In each case, we will not attempt to develop a non-standard QM in detail, but instead content ourselves with considering simple representative examples.

(a) *Non-complete measurements or non-complete Schrödinger evolution.* Let us consider a QM variant that allows a non-tracepreserving (and hence non-unitary) but invertible single-qubit operation of the form:

$$G = \begin{pmatrix} 1 & 0 \\ 0 & 1 + \epsilon \end{pmatrix}, \quad (1)$$

where $\epsilon > 0$ is a real number. The resultant state $\sum_x \alpha_x |x\rangle$ must be normalized by dividing it by the normalization factor $\sqrt{\sum_x |\alpha_x|^2}$ immediately before a measurement, making measurements nonlinear. Given the entangled state $(1/\sqrt{2})(|01\rangle + |10\rangle)$ that Alice and Bob share, to transmit a superluminal signal, Alice applies either G^m (where $m \geq 1$ is an integer) or the identity operation I to her qubit. Bob's particle is left, respectively, in the state $\rho_B^{(1)} \propto \frac{1}{2}(|0\rangle\langle 0| + (1 + \epsilon)^{2m}|1\rangle\langle 1|)$ or $\rho_B^{(0)} = \frac{1}{2}(|0\rangle\langle 0| + |1\rangle\langle 1|)$, which can in principle be distinguished, the distance between the states being greater for larger m (cf. Section II B), leading to a superluminal signal from Alice to Bob.

More generally, we may allow non-unitary and irreversible evolution but still conform to no-signaling, provided the corresponding set of operator(s) is *complete*, i.e., constitutes a partition of unity. Suppose Alice and Bob share the state ρ_{AB} , and Alice evolves her part of ρ_{AB} locally through the linear operation given by the set \mathcal{P} of (Kraus) operator elements $\{E_j \equiv e_j \otimes \mathbb{I}_B, j = 1, 2, 3, \dots\}$ [10], where \mathbb{I}_B is the identity operator in Bob's subspace. Bob's reduced density operator ρ'_B conditioned on her performing the operation and after normalization is:

$$\rho'_B = \mathcal{N}^{-1} \text{Tr}_A \left[\sum_j E_j \rho_{AB} E_j^\dagger \right] = \mathcal{N}^{-1} \text{Tr}_A \left[\sum_j E_j^\dagger E_j \rho_{AB} \right], \quad \mathcal{N} = \text{Tr}_{AB} \left[\sum_j E_j^\dagger E_j \rho_{AB} \right], \quad (2)$$

where \mathcal{N} is the normalization factor. We satisfy the no-signaling condition $\rho'_B = \rho_B$ only if ρ_{AB} is unentangled or \mathcal{P} satisfies the completeness relation

$$\sum_j e_j^\dagger e_j = \mathbb{I}_A, \quad (3)$$

which guarantees that the operation preserves norm \mathcal{N} . Here \mathbb{I}_A is the identity operator in Alice's subspace. If the norm is not preserved, renormalization is required, making the evolution effectively nonlinear. If the system A is subjected to unitary evolution or non-unitary evolution due to noise, or to standard projective measurements or more general measurements described by positive operator valued measures, the corresponding map satisfies Eq. (3), and $\rho'_B = \rho_B$. For terminological brevity, we call a (non-standard) gate like G , or a non-complete operation \mathcal{P} that enables superluminal signaling, as 'superluminal gate', and denote the set of all superluminal gates by ' $C^<$ '. For the purpose of this work, $C^<$ is restricted to qubit or qutrit gates. Non-unitary super-quantum cloning or deleting, introduced in Ref. [31], which lead to superluminal signaling, are other examples of non-complete operations.

Even at the single-particle level, if the measurement is non-complete, there is a superluminal signaling due to breakdown in non-contextuality coming from the renormalization. As a simple illustration, suppose we are given two

observers Alice and Bob sharing a delocalized qubit, $\cos(\theta/2)|0\rangle + \sin(\theta/2)|1\rangle$, with eigenstate $|1\rangle$ localized near Alice and $|0\rangle$ near Bob. With an m -fold application of G (which can be thought of as an application of imaginary phase on Alice's side, leading to selective augmentation of amplitude) on this state, Alice produces the (unnormalized) state $\cos(\theta/2)|0\rangle + (1 + \epsilon)^m \sin(\theta/2)|1\rangle$, so that after renormalization, Bob's probability of obtaining $|0\rangle$ has changed in a context-dependent fashion from $\cos^2(\theta/2)$ to $\cos^2(\theta/2)(\cos^2(\theta/2) + (1 + \epsilon)^{2m} \sin^2(\theta/2))^{-1}$. By thus nonlocally controlling the probability with which Bob finds $|0\rangle$, Alice can probabilistically signal Bob superluminally.

(b) *Nonlinear evolution.* As a simple illustration of a superluminal gate arising from nonlinear evolution, we consider the action of the nonlinear two-qubit 'OR' gate R , whose action in a preferred (say, computational) basis is given by:

$$\left. \begin{array}{l} |00\rangle \pm |11\rangle \\ |01\rangle \pm |10\rangle \\ |01\rangle \pm |11\rangle \end{array} \right\} \xrightarrow{R} |01\rangle \pm |11\rangle; \quad \begin{array}{l} |00\rangle \pm |10\rangle \xrightarrow{R} |00\rangle \pm |10\rangle, \\ |\alpha\beta\rangle \xrightarrow{R} |\alpha\beta\rangle. \end{array} \quad (4)$$

If the two qubits are entangled with other qubits, then the gate is assumed to act in each subspace labelled by states of the other qubits in the computational basis. Alice and Bob share the entangled state $|\Psi\rangle = 2^{-1/2}(|00\rangle - |11\rangle)$. To transmit a bit superluminally Alice measures her qubit in the computational basis or the diagonal basis $\{|\pm\rangle \equiv 2^{-1/2}(|0\rangle \pm |1\rangle)\}$, leaving Bob's qubit's density operator in a computational basis ensemble or a diagonal basis ensemble, which are equivalent in standard QM. However, with the nonlinear operation R , the two ensembles can be distinguished. Bob prepares an ancillary qubit in the state $|0\rangle$, and applies a CNOT on it, with his system qubit as the control. On the resulting state he performs the nonlinear gate R , and measures the ancilla. The computational (resp., diagonal) basis ensemble yields the value 1 with probability $\frac{1}{2}$ (resp., 1). By a repetition of the procedure a fixed number m of times, a superluminal signal is transmitted from Alice to Bob with exponentially small uncertainty in m . Analogous to Eq. (4), one can define a 'nonlinear AND', which, again, similarly leads to a nonlocal signaling.

(c) *Departure from the Born $|\psi|^2$ probability rule.* Gleason's theorem shows that the Born probability rule that identifies $|\psi|^2$ as a probability measure, and more generally, the trace rule, is the only probability prescription consistent in 3 or larger dimensions with the requirement of non-contextuality [4]. Suppose we retain unitary evolution, which preserve the 2-norm, but assume that the probability of a measurement on the state $\sum_j \alpha_j |j\rangle$ is of the form $|\alpha_j|^p / \sum_k |\alpha_k|^p$ for outcome j , and p any non-negative real number. The renormalization will make the measurement contextual, giving rise to a superluminal signal. One might consider more general evolution that preserves a p -norm, but there are no linear operators that do so except permutation matrices [14].

For example, let Alice and Bob share the two-qubit entangled state $\cos\theta|00\rangle + \sin\theta|11\rangle$ ($0 < \theta < \pi/2$). The probability for Alice measuring her particle in the computational basis and finding $|0\rangle$ (resp., $|1\rangle$) must be the same as that for a joint measurement in this basis to yield $|00\rangle$ (resp., $|11\rangle$). Therefore Bob's reduced density operator is given by the state $\rho^{(1)} = (\cos^p\theta|0\rangle\langle 0| + \sin^p\theta|1\rangle\langle 1|)/(\cos^p\theta + \sin^p\theta)$. On the other hand, if Alice employs an ancillary, third qubit prepared in the state $|0\rangle$, and applies a Hadamard on it conditioned on her qubit being in the state $|0\rangle$, she produces the state $\frac{\cos\theta}{\sqrt{2}}|000\rangle + \frac{\cos\theta}{\sqrt{2}}|001\rangle + \sin\theta|110\rangle$. The probability that Alice obtains outcomes 00, 01 or 10 must be that for a joint measurement to yield 000, 001 or 110. Along similar lines as in the above case we find that she leaves Bob's qubit in the state

$$\rho^{(2)} \equiv \frac{2^{(1-p/2)} \cos^p\theta |0\rangle\langle 0| + \sin^p\theta |1\rangle\langle 1|}{2^{(1-p/2)} \cos^p\theta + \sin^p\theta}. \quad (5)$$

Since $\rho^{(1)}$ and $\rho^{(2)}$ are probabilistically distinguishable, with sufficiently many shared copies Alice can signal Bob one bit superluminally, unless $p = 2$.

B. Exponential gates

As superluminal quantum gates like G or R are internally consistent, one can consider why no such operation occurs in Nature, whether a fundamental principle prevents their physical realization. One candidate principle is of course no-signaling itself. Alternatively, since we would like to derive it, linearity of QM may be taken as an axiom. Since all the above non-standard operations involve an overall nonlinear evolution, the assumption of strict quantum mechanical linearity can indeed rule out such non-standard gates. Yet it must be admitted that, from a purely physics viewpoint, assuming that QM is linear affords no greater insight than assuming it to be a non-signaling theory. We would like to suggest that the the absence of such operations may have a complexity theoretic basis.

Both superluminal gates as well as hypothetical gates that allow efficient solving of intractable problems involve some sort of communication across superposition branches. In particular, the superluminal gates of Section II A can be turned into the latter type of gates, as discussed below.

(a) *Non-complete quantum gates.* It is easily seen that the gate G in Eq. (1) can be used to solve **NP**-complete problems efficiently. Consider solving boolean satisfiability (SAT), which is **NP**-complete: given an efficiently computable black box function $f : \{0, 1\}^n \mapsto \{0, 1\}$, to determine if there exists x such that $f(x) = 1$. With the use of an oracle that computes $f(x)$, we prepare the $(n + 1)$ -qubit entangled state

$$|\Psi_{nc}\rangle = 2^{-n/2} \sum_{x \in \{0, 1\}^n} |x\rangle |f(x)\rangle, \quad (6)$$

and then apply G^m to the second, 1-qubit register, where m is a sufficiently large integer, before measuring the register. In particular, suppose that at most one solution exists. The un-normalized ‘probability mass’ of obtaining outcome $|1\rangle$ becomes 1 (and the normalized probability about $1/2$) when $m = n/(2 \log(1 + \epsilon))$, if there is a solution, or, if no solution exists, remains 0. Repeating the experiment a fixed number of times, and applying the Chernoff bound, we find that to solve SAT, we only require $m \in O(n)$. For terminological brevity, we will call as ‘exponential gate’ such a non-standard gate that enables the efficient computation of **NP**-complete problems, and denote by E the set of all exponential gates, restricted in the present work to qubits and qutrit gates.

(b) *Nonlinear quantum gates.* The nonlinear operation R in Eq. (4) can be used to efficiently simulate nondeterminism. We prepare the state $|\psi\rangle$ in Eq. (6), where the first n qubits are called the ‘index’ qubits and the last one the ‘flag’ qubit. There are 2^{n-1} 4-dim subspaces, consisting of the first index qubit and the flag qubit, labelled by the index qubits $2, \dots, n$. On each such subspace, the first index qubit and flag qubit are in one of the states $|00\rangle + |11\rangle$, $|01\rangle + |10\rangle$, $|00\rangle + |10\rangle$. The operation Eq. (4) is applied n times, pairing each index qubit sequentially with the flag. The number of terms with 1 on the flag bit doubles with each operation so that after the n operations, it becomes disentangled and can then be read off to obtain the answer [16]. A slight modification of this algorithm solves **#P**-complete problems efficiently, by replacing the flag qubit with $\log n$ qubits and the 1-bit nonlinear OR operation with the corresponding nonlinear counting. The final readout is then the number of solutions to $f(x) = 1$ [16]. Applying the nonlinear OR and AND alternatively to the state $|\psi\rangle$ in Eq. (6) allows one to efficiently solve the quantified Boolean formula problem, which is **PSPACE**-complete [32].

(c) *Non-Gleasonian gates.* By employing polynomially many ancillas in the method of (c) in the previous subsection, one can show that non-Gleasonian quantum computers (for which $p \neq 2$) can solve **PP**-complete problems [33] efficiently. Defining **BQP** _{p} as similar to **BQP**, except that the probability of measuring a basis state $|x\rangle$ equals $|\alpha_x|^p / \sum_y |\alpha_y|^p$ (so that **BQP**₂ = **BQP**), it can be shown that **PP** \subseteq **BQP** _{p} for all constants $p \neq 2$, and that, in particular, **PP** exactly characterizes the power of a quantum computer with even-valued p (except $p = 2$) [14].

In view of the connection between the two classes of gates, we now propose, as we earlier did in Ref. [8], that the reason for the absence in Nature of the superluminal gates of Section II A is WNHE: in a universe that is a polynomial place, exponential gates like G and R are ruled out. In the next Section we will consider in further detail the viability of the WNHE assumption as an explanation for no-signaling.

C. Polynomial superluminal gates

Even though WNHE excludes the type of superluminal gates considered above, for the exclusion to be general, it would have to be shown that every superluminal gate is exponential, i.e., $C^< \subseteq E$. It turns out that this cannot be done, because one can construct hypothetical *polynomial superluminal gates*, which are superluminal operations that are not exponential. In fact, it is probably true that $E \subset C^<$. To see this, let us consider solving the **NP**-complete problem associated with Eq. (6) via Grover search [34], which is optimal for QM [35] but offers only a quadratic speed-up, thus leaving the complexity of the problem exponential in n , at least in the black box setting. The optimality proof relies on showing that, given the problem of distinguishing an empty oracle ($\forall_x A(x) = 0$) and a non-empty oracle containing a single random unknown string y of known length n (i.e. $A(y) = 1$, but $\forall_{x \neq y} A(x) = 0$), subject to the constraint that its overall evolution be unitary, and linear (so that in a computation with a nonempty oracle, all computation paths querying empty locations evolve exactly as they would for an empty oracle), the speed-up over a classical search is at best quadratic.

Any degree of amplitude amplification of the marked state above the quadratic level would then require empty superposition branches being ‘made aware’ of the presence of a non-empty branch, i.e., a nonlinearity of some sort. Let us suppose Bob can perform a trace-preserving nonlinear transformation $\rho_j \rightarrow \tilde{\rho}_j$ of the above kind on an unknown ensemble of separable states. Further, let Alice and Bob share an entangled state, by which Alice is able to prepare, employing two different positive operator-valued measures (POVMs), two different but equivalent ensembles of Bob. Then, depending on Alice’s choice, his reduced density matrix evolves as $\rho_B = \sum_j p_j \rho_j \rightarrow \sum_j p_j \tilde{\rho}_j \equiv \rho'$ or $\rho_B = \sum_s p_k \rho_k \rightarrow \sum_k p_k \tilde{\rho}_k \equiv \rho''$ where (ρ_j, p_j) and (ρ_k, p_k) are distinct, equivalent ensembles [36]. The assumption of linearity is sufficient to ensure that $\rho' = \rho''$. This is not guaranteed in the presence of nonlinearity, leading to a

potential superluminal signal. In a nonlinearity of the above kind, the result would depend on whether the particular ensemble remotely prepared by Alice has states that include $|y\rangle$ in the superposition. This would lead to a scenario similar to that encountered with nonlinear gate R in Section II A.

Possibly the simplest examples of polynomial superluminal gates are the non-invertible *constant gates*, which map any state in an input Hilbert space to a fixed state in the output Hilbert space, and have the form $|\xi\rangle \otimes \sum_j |j\rangle$, for some fixed ξ . Examples in matrix notation are:

$$Q = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}; \quad Q' = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (7)$$

acting in Hilbert space $\mathcal{H}_2 \equiv \text{span}\{|0\rangle, |1\rangle\}$ and $\mathcal{H}_3 \equiv \text{span}\{|0\rangle, |1\rangle, |2\rangle\}$, respectively. They have the effect of mapping any input state in \mathcal{H}_2 to a fixed (apart from a normalization factor) state $|\xi\rangle$, in this case $|\xi\rangle$ being $|0\rangle$. In Eq. (7), we do not in general require the input and output bases to be the same, nor indeed that the input and output Hilbert subspaces be the same (for example, as with the distinct incoming and outgoing modes of a scattering problem.)

Both Q and Q' are non-complete, inasmuch as $Q^\dagger Q \neq \mathbb{I}$ and $(Q')^\dagger Q' \neq \mathbb{I}$, and represent superluminal gates. For example, by applying or not applying Q to her register in the state $(1/\sqrt{2})(|01\rangle + |10\rangle)$ shared with Bob, Alice can remotely prepare his state to be the pure state $(1/\sqrt{2})(|0\rangle + |1\rangle)$ or leave it as a maximal mixture, respectively, corresponding to a superluminal signal of about 0.3 bits (determined by the Holevo bound). Similarly, by choosing to apply, or not, Q' on her half of the state $(1/\sqrt{2})(|11\rangle + |22\rangle)$ shared with Bob, Alice can superluminally signal him.

The constant gate is linear and presumes no re-normalization following its non-complete action. The probability of the occurrence of a constant gate C when it is applied to a state $|\psi\rangle$ is simply given by $\|C|\psi\rangle\|^2$, per the standard prescription. One consequence is that it could not be used to violate no-signaling without the use of entanglement. As an illustration: in \mathcal{H}_3 , let the states $|0\rangle$ and $|1\rangle$ be localized near Alice and $|2\rangle$ near Bob. Applying Q' on the state $|\psi\rangle \equiv a|0\rangle + b|1\rangle + c|2\rangle$, Alice obtains the (unnormalized) state $Q'|\psi\rangle = (a+b)|0\rangle + c|2\rangle$. If renormalization were allowed, Alice could contextually (i.e., nonlocally) influence Bob's probability to find $|2\rangle$ to be $|c|^2/(|a+b|^2 + |c|^2)$ or $|c|^2$, depending on whether she applies Q' or not. The linearity of the constant gate requires the interpretation that following her application of Q' , Alice can detect the particle with probability $|a+b|^2$, while for Bob, the probability remains $|c|^2$. Though non-complete operations do not necessarily conserve probability, still, as we will find below and later that in situations of interest they can exactly or effectively conserve probability.

On the other hand, neither Q nor Q' nor a general constant gate is an exponential gate: each of them simply transforms any valid input into a fixed output. Intuitively, this lack of any dependence on the input clearly limits its computational power. Operations Q and Q' in Eq. (7) can be extended to a more general class of polynomial superluminal operations acting on qubits, qutrit and higher dimensional qudits, such as:

$$Q_2(\phi) = \begin{pmatrix} 1 & e^{i\phi} \\ 0 & 0 \end{pmatrix}, \quad Q_3(\phi_1, \phi_2) = \begin{pmatrix} 1 & e^{i\phi_1} & e^{i\phi_2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \text{ etc.} \quad (8)$$

By definition, $Q = Q_2(0)$ and $Q' = Q_3(0, 0)$. To see that $Q_2(\phi)$ is a polynomial operation, it suffices to show that it can be simulated using only polynomial amount of standard quantum mechanical resources, which we do in the following theorem. Let \mathbf{BQP}_c denote the complexity class of problems that can be efficiently solved on a standard quantum computer that can access a constant gate. Then:

Theorem 1 $\mathbf{BQP}_c = \mathbf{BQP}$.

Proof. It is clear that any problem in \mathbf{BQP} can be efficiently solved using resources of \mathbf{BQP}_c , by simply not using the constant gates. Now let us consider the simulation the other way. Given an arbitrary $(n+1)$ -qubit state $|\psi\rangle = |\alpha\rangle|0\rangle + |\beta\rangle|1\rangle$, where the n -qubit states $|\alpha\rangle$ and $|\beta\rangle$ are neither necessarily mutually orthogonal nor normalized and with $\| |\alpha\rangle \|^2 + \| |\beta\rangle \|^2 = 1$, the action of $Q_2(\phi)$ on the last qubit is to produce $Q_2|\psi\rangle = (|\alpha\rangle + e^{i\phi}|\beta\rangle)|0\rangle \equiv |\psi'\rangle|0\rangle$, which is interpreted as $\mathcal{N} \equiv \langle \psi' | \psi' \rangle = 1 + 2[\cos(\phi)\Re(\langle \alpha | \beta \rangle) - \sin(\phi)\Im(\langle \alpha | \beta \rangle)]$ copies of the normalized state $|\underline{\psi'}\rangle \equiv |\psi'\rangle/\sqrt{\mathcal{N}}$ and a copy of $|0\rangle$. If $|\alpha\rangle$ and $|\beta\rangle$ are mutually orthogonal, and thus the reduced density operator for the last qubit is diagonal in the computational basis, then $\mathcal{N} = 1$, and no such special interpretation is needed.

To simulate the production of $|\psi'\rangle$ with standard quantum resources, one first applies a phase gate $\begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$ followed by a Hadamard on the last qubit, to obtain the state $(1/\sqrt{2})(|\psi'\rangle|0\rangle + |\psi''\rangle|1\rangle)$ where $|\psi''\rangle = |\alpha\rangle - e^{i\phi}|\beta\rangle$. Measurement on the last qubit in the computational basis yields $|0\rangle$, and hence $|\underline{\psi'}\rangle$ in the first register, with probability $\| |\psi'\rangle/\sqrt{2} \|^2 = \mathcal{N}/2$, which is to say that the simulation of Q_2 succeeds with probability $1/2$, irrespective of n . Similar

arguments hold for Q_3 , etc. Therefore, the class of problems efficiently solvable with standard quantum computation augmented by the non-standard family of constant gates is in **BQP**. ■

It is worth noting that the constant gate is quite different from the following two operations that appear to be similar, but are in fact quite distinct. The first operation is a standard quantum mechanical CP map, polynomial and not superluminal; the second is exponential and consequently superluminal.

(a) To begin with, a constant gate is not a quantum deleter [37], in which a qubit is subjected to a *complete* operation, in specific, a contractive CP map that prepares it asymptotically in a fixed state $|0\rangle$. The action of a quantum deleter is given by an amplitude damping channel [10], which has an operator sum representation, respectively

$$\rho_2 \longrightarrow \sum_j E_j \rho_2 E_j^\dagger; \quad \rho_3 \longrightarrow \sum_j E'_j \rho_3 E_j'^\dagger, \quad (9)$$

in the qubit case or when extended to the qutrit case, with the Kraus operators given by Eq. (10a) or (10b), respectively

$$E_1 \equiv \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad E_2 \equiv \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad (10a)$$

$$E'_1 \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad E'_2 \equiv \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad E'_3 \equiv \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (10b)$$

Unlike in the case of Q , Q' or Q'' , there is no actual destruction of quantum information, but its transfer through dissipative decoherence into correlations with a zero-temperature environment. The reduced density operator of Bob's entangled system remains unaffected by Alice's application of this operation on her system. The deleting action, though nonlinear at the state vector level, nevertheless acts linearly on the density operator.

(b) Next we note that the constant gate is quite different from the 'post-selection' operation, which is a *deterministic* rank-1 projection [14]. Verbally, if the constant gate corresponds to the operation "for all input states $|j\rangle$ in the computational basis, set the output state to $|\xi\rangle$, independently of j , except for a global phase", where $|\xi\rangle$ is some fixed state, then post-selection corresponds to the action "for all input states $|j\rangle$, if $j \neq \xi$, then discard branch $|j\rangle$ ". Post-selective equivalents of Q and Q' are

$$Q_{\text{PS}} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \quad Q'_{\text{PS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (11)$$

followed by renormalization. In particular, whereas the action of Q on the first of two particles in the state $(1/\sqrt{2})(|00\rangle + |11\rangle)$ leaves the second particle in the state $(1/\sqrt{2})(|0\rangle + |1\rangle)$, that of Q_{PS} leaves the second particle in the state $|0\rangle$. It is straightforward to see that post-selection is an exponential operation: acting it on the second qubit of $|\Psi_{\text{nc}}\rangle$ in Eq. (6), and post-selecting on 1, we obtain the solution to SAT in one time-step.

The seemingly immediate conclusion due to the fact $C^< \not\subseteq E$ is that the WNHE assumption is not strong enough to derive no-signaling, and would have to be supplemented with additional assumption(s), possibly purely physically motivated ones, prohibiting the physical realization of polynomial superluminal gates.

An alternative, highly unconventional reading of the situation is that WNHE is a fundamental principle of the physical world, while the no-signaling condition is in fact not universal, so that some polynomial superluminal gates may actually be physically realizable. Quite surprisingly, we may be able to offer some support for this viewpoint. We believe that constant gates of above type can be quantum optically realized when a photon detection is made at a *path singularity*, defined as a point in space where two or more incoming paths converge and terminate. In graph theoretic parlance, a path singularity is a terminal node in a directed graph, of degree greater than 1.

We describe in Section III A an experiment that possibly physically realizes Q . In principle, a detector placed at the focus of a convex lens realizes such a path singularity. This is because the geometry of the ray optics associated with the lens requires rays parallel to the lens axis to converge to the focus after refraction, while the destructive nature of photon detection implies the termination of the path. Although conceptually and experimentally simple, the high degree of mode filtering or spatial resolution that the experiment requires will be the main challenge in implementing it. Indeed, we believe this is the reason that such gates have remained undiscovered so far.

Our argument here has implicitly assumed that $\mathbf{P} \neq \mathbf{NP}$. If it turns out that $\mathbf{P} = \mathbf{NP}$, then even the obviously non-physical operations such as G or R would be polynomial gates, and the WNHE assumption would not be able to exclude them. Nevertheless, the question of existence and testability of certain superluminal gates, which is the main result of this work, would still remain valid and of interest. If polynomial superluminal gates are indeed found

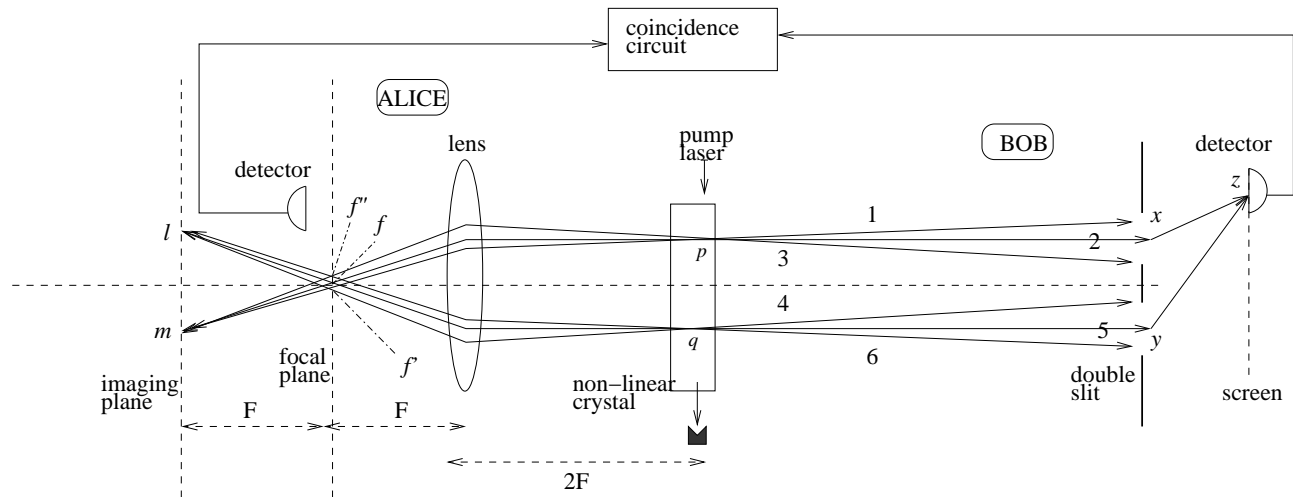


FIG. 1: An ‘unfolded’ version of the Innsbruck experiment (not to scale). A pair of momentum-entangled photons is created by type-I parametric down conversion of the pump laser. Alice’s photon (the signal photon) is registered by a detector behind a lens. Bob’s photon (the idler) is detected behind a double-slit assembly. If her detector is placed in the focal plane of the lens (of focal length F), it projects Bob’s state into a mixture of plane waves, which produce an interference pattern on Bob’s screen *in coincidence* with any fixed detection point on Alice’s focal plane. Bob’s pattern in his *singles count*, being the integration of such patterns over all focal plane points, shows no interference pattern. On the other hand, positioning her detector in the imaging plane can potentially reveal the path the idler takes through the slit assembly, and thus does not lead to an interference pattern on Bob’s screen even in the coincidence counts.

to exist (and given that other superluminal gates do not seem to exist anyway), this would give us greater confidence that $\mathbf{P} \neq \mathbf{NP}$ (or, to be safe, that even Nature does not ‘know’ that $\mathbf{P} = \mathbf{NP}$!) and that the assumption of WNHE is indeed a valid and fruitful one.

III. PART II: QUANTUM OPTICAL TEST

A. An experiment with entangled pairs of photons

Our proposed implementation of Q' , based on the use of entanglement, is broadly related to the type of quantum optical experiments encountered in Refs. [38], and closely related to an experiment performed in Innsbruck that elegantly illustrates wave-particle duality by means of entangled light [39, 40]. In the Innsbruck experiment, pairs of position-momentum entangled photons are produced by means of type-I spontaneous parametric down-conversion (SPDC) at a nonlinear source, such as a BBO crystal. The two outgoing conical beams from the nonlinear source are presented ‘unfolded’ in Figure 1. One of each pair, called the ‘signal photon’, is received by Alice, while the other, called the ‘idler’, is received and analyzed by Bob. Alice’s photon is registered by a detector behind a lens.

Bob’s photon is detected after it enters a double-slit assembly. If Alice’s detector, which is located behind the lens, is positioned at the focal plane of the lens and detects a photon, it localizes Alice’s photon to a point on the focal plane. By virtue of entanglement, this projects the state of Bob’s photon to a ‘momentum eigenstate’, a plane wave propagating in a particular direction. For example, if Alice detects her photon at f , f' or f'' , Bob’s photon is projected to a superposition of the parallel modes 2 and 5, modes 1 and 4, or modes 3 and 6. Since this cannot reveal positional information about whether the particle originated at p or q , and hence reveals no which-way information about slit passage, therefore, *in coincidence* with a registration of her photon at a focal plane point, the idler exhibits a Young’s double-slit interference pattern [39, 40]. The patterns corresponding to Alice’s registering her photon at f , f' or f'' will be mutually shifted. Bob’s observation in his single counts will therefore not show any sign of interference, being the average of all possible such mutually shifted patterns. The interference pattern is seen by Bob in coincidence with Alice’s detection.

If her detector is placed at the imaging plane (at distance $2F$ from the plane of the lens), a click of the detector can reveal the path the idler takes from the crystal through the slit assembly which therefore cannot show the interference pattern even in the coincidence counts. For example, if Alice detects her photon at l (resp., m), Bob’s photon is projected to a superposition of the mutually non-parallel modes 4, 5 and 6 (resp., 1, 2 and 3) and, because the double-slit assembly is situated in the near field, can then enter only slit y (resp., x). Therefore, Alice’s imaging plane

measurement gives path or position information of the idler photon, so that no interference pattern emerges in Bob's coincidence counts [39, 40], and consequently also in his singles counts. This qualitative description of the Innsbruck experiment is made quantitative using a simple six-mode model in the next Subsection.

1. Quantum optical description of the Innsbruck experiment

Here the state of the SPDC field of Figure 2 is modeled by a 6-mode vector:

$$|\Psi\rangle = (1 + \frac{\epsilon}{\sqrt{6}} \sum_{j=1}^6 a_j^\dagger b_j^\dagger) |\text{vac}\rangle \quad (12)$$

where $|\text{vac}\rangle$ is the vacuum state, a_j^\dagger (resp., b_j^\dagger) are the creation operators for Alice's (resp., Bob's) light field on mode j , per the mode numbering scheme in Figure 2. The quantity ϵ ($\ll 1$) depends on the pump field strength and the crystal nonlinearity. The coincidence counting rate for corresponding measurements by Alice and Bob is proportional to the square of the second-order correlation function, and given by:

$$R_\alpha(z) \propto \langle \Psi | E_z^{(-)} E_\alpha^{(-)} E_\alpha^{(+)} E_z^{(+)} | \Psi \rangle = \| E_\alpha^{(+)} E_z^{(+)} | \Psi \rangle \|^2, \quad (\alpha = f, f'', l, m, \dots). \quad (13)$$

where $E_\alpha^{(+)}$ represents the positive frequency part of the electric field at a point on Alice's focal or imaging plane, and $E_z^{(+)}$ that of the electric field at an arbitrary point z on Bob's screen. We have:

$$E_z^{(+)} = e^{ikr_D} (e^{ikr_2} \hat{b}_2 + e^{ikr_5} \hat{b}_5) + e^{ikr_{D'}} (e^{ikr_1} \hat{b}_1 + e^{ikr_4} \hat{b}_4) + e^{ikr_{D''}} (e^{ikr_3} \hat{b}_3 + e^{ikr_6} \hat{b}_6), \quad (14)$$

where k is the wavenumber, r_D the distance from the EPR source to the upper/lower slit on Bob's double slit diaphragm (the length of the segment \overline{qy} or \overline{px}); r_2 (resp., r_5) is the distance from the lower (resp., upper) slit to z . The other two terms in Eq. (14), pertaining to the other two pair of modes, are obtained analogously. We study the two cases, corresponding to Alice making a remote position or remote momentum measurement on the idler photons.

Case 1. Alice remotely measures position (path) of the idler. Suppose Alice positions her detector at the imaging plane and detects a photon at l or m . The corresponding field at her detector is

$$E_m^{(+)} = e^{iks_m} (\hat{a}_1 + \hat{a}_2 + \hat{a}_3); \quad E_l^{(+)} = e^{iks_l} (\hat{a}_4 + \hat{a}_5 + \hat{a}_6), \quad (15)$$

where s_m (resp., s_l) is the path length along any ray path from the source point p (resp., q) through the lens upto image point m (resp., l). By Fermat's principle, all paths connecting a given pair of source and image point are equal. Setting $\alpha = l, m$ in Eq. (13), and substituting Eqs. (12), (14) and (15) in Eq. (13), we find the coincidence counting rate for detections by Alice and Bob to be

$$R_m(z) \propto \epsilon^2 |e^{ikr_1} + e^{ikr_2} + e^{ikr_3}|^2; \quad R_l(z) \propto \epsilon^2 |e^{ikr_4} + e^{ikr_5} + e^{ikr_6}|^2, \quad (16)$$

which is essentially a single slit diffraction pattern formed behind, respectively, the upper and lower slit. The intensity pattern Bob finds on his screen in the singles count, obtained by averaging $R_\alpha(z)$ over $\alpha = l, m$, is thus not a double-slit interference pattern, but an incoherent mixture of the two single slit patterns. A similar lack of interference pattern is obtained by Bob if Alice makes no measurement.

Case 2. Alice remotely measures momentum (direction) of the idler. Alice positions her detector on the focal plane of the lens. If she detects a photon at f , f' or f'' , the field at her detector is, respectively,

$$E_f^{(+)} = e^{ikr_{2f}} \hat{a}_2 + e^{ikr_{5f}} \hat{a}_5 = e^{ikr_f} (\hat{a}_2 + \hat{a}_5), \quad (17a)$$

$$E_{f'}^{(+)} = e^{ikr_{1f'}} \hat{a}_1 + e^{ikr_{4f'}} \hat{a}_4 = e^{ikr_{1f'}} (\hat{a}_1 + e^{ik(r_{5f'} - r_{1f'})} \hat{a}_4), \quad (17b)$$

$$E_{f''}^{(+)} = e^{ikr_{3f''}} \hat{a}_3 + e^{ikr_{6f''}} \hat{a}_6 = e^{ikr_{3f''}} (\hat{a}_3 + e^{ik(r_{6f''} - r_{3f''})} \hat{a}_6), \quad (17c)$$

where r_{2f} (resp., r_{5f}) is the distance from p (resp., q) along the path 2 (resp., 5) path through the lens upto point f . The distances along the two paths being identical, $r_{2f} = r_{5f} \equiv r_f$. The distances $r_{1f'}$, $r_{4f'}$, $r_{3f''}$ and $r_{6f''}$ are defined analogously. Substituting Eqs. (12), (14) and (17) in Eq. (13), we find the coincidence counting rate is given by

$$R_f(z) \propto \epsilon^2 [1 + \cos(k \cdot [r_2 - r_5])], \quad (18a)$$

$$R_{f'}(z) \propto \epsilon^2 [1 + \cos(k \cdot [r_1 - r_4] + \omega_{14})], \quad (18b)$$

$$R_{f''}(z) \propto \epsilon^2 [1 + \cos(k \cdot [r_3 - r_6] + \omega_{36})], \quad (18c)$$

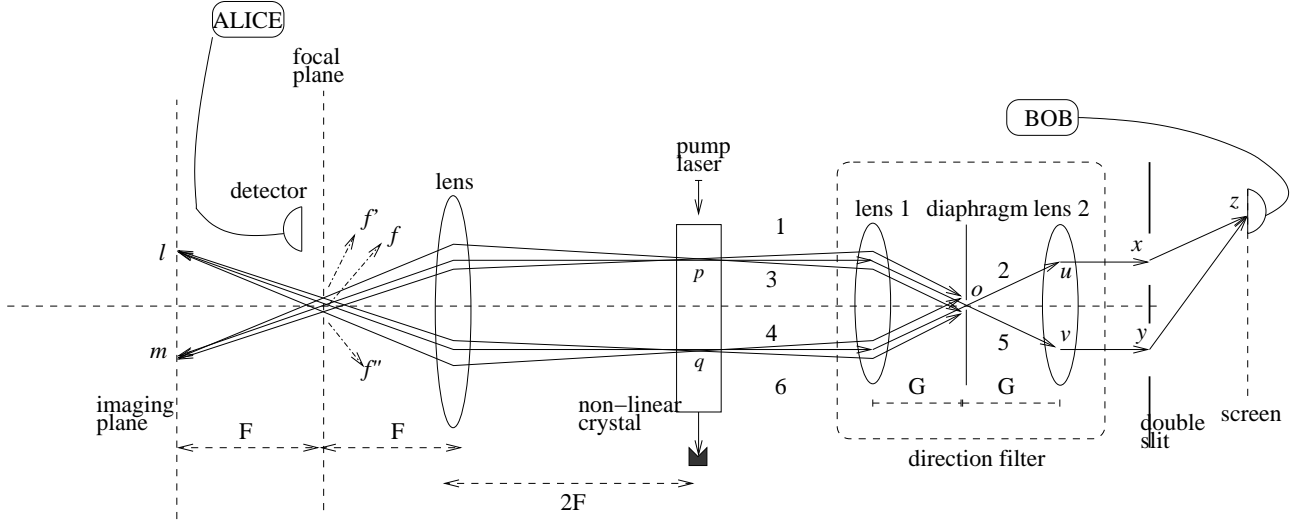


FIG. 2: The modified Innsbruck experiment (not to scale): Same configuration as in Figure 1, except that Bob's photon (the idler), before entering the double-slit assembly, traverses a direction filter that permits only (nearly) horizontal modes to pass through, absorbing the other modes at the filter walls. The direction filter acts as a state filter that ensures that Bob receives only the *pure* state consisting of the horizontal modes. Thus if Alice makes no measurement or makes a detection at f , Bob's corresponding photon builds an interference pattern of the modes 2 and 5 in the *singles* counts. On the other hand, if Alice positions her detector in the imaging plane, she knows the path the idler takes through the slit assembly. Thus no interference pattern is found on Bob's screen even in the coincidence counts.

where $\omega_{14} \equiv k(r_{4f} - r_{1f})$ and $\omega_{36} \equiv k(r_{6f} - r_{3f})$ are fixed for a given point on the focal plane. Each equation in Eq. (18) represents a conventional Young's double slit pattern. Conditioned on Alice detecting photons at f , Bob finds the pattern $R_f(z)$, and similarly for points f' and f'' . In his singles count, Bob perceives no interference, because he is left with a statistical mixture of the patterns (18a), (18b), (18c), etc., corresponding to *all* points on Alice's focal plane illuminated by the signal beam.

2. The proposed experiment

The experiment proposed here, presented earlier by us in Ref. [41], is derived from the Innsbruck experiment, and therefore called 'the Modified Innsbruck experiment'. It was claimed to manifest superluminal signaling, though it was not clear what the exact origin of the signaling was, and in particular, which assumption that goes to proving the no-signaling theorem was being given up. The Modified Innsbruck experiment is revisited here in order to clarify this issue in detail in the light of the discussions of the previous Sections. This will help crystallize what is, and what is not, responsible for the claimed signaling effect. In Ref. [42], we studied a version of nonlocal communication inspired by the original Einstein-Podolsky-Rosen thought experiment [1]. Recently, similar experiments, also based on the Innsbruck experiment, have been independently proposed in Refs. [43, 44].

First we present a qualitative overview of the modified Innsbruck experiment. The only material difference between the original Innsbruck experiment and the modified version we propose here is that the latter contains a 'direction filter', consisting of two convex lenses of the same focal length G , separated by distance $2G$. Their shared focal plane is covered by an opaque screen, with a small aperture o of diameter δ at their shared focus. We want δ to be small enough so that only almost horizontal modes are permitted by the filter to fall on the double slit diaphragm. The angular spread (about the horizontal) of the modes that fall on the aperture is given by $\Delta\theta = \delta/G$, we require that $(\delta/G)\sigma \ll \lambda$, where σ is the slit separation, to guarantee that only modes that are horizontal or almost horizontal are selected to pass through the direction filter, to produce a Young's double-slit interference pattern on his screen plane. On the other hand, we don't want the aperture to be so small that it produces significant diffraction, thus: $\delta \gg \lambda$. Putting these conditions together, we must have

$$1 \ll \frac{\delta}{\lambda} \ll \frac{G}{\sigma}. \quad (19)$$

The ability to satisfy this condition, while preferable, is not crucial. If it is not satisfied strictly, the predicted signal is weaker but not entirely suppressed. The point is clarified further down.

If Alice makes no measurement, the idler remains entangled with the signal photon, which renders incoherent the beams coming through the upper and lower slits on Bob's side, so that he will find no interference pattern on his screen. Similarly, if she detects her photon in the imaging plane, she localizes Bob's photon at his slit plane, and so, again, no interference pattern is seen. Thus far, the proposed experiment has the same effect as the Innsbruck experiment.

On the other hand, if Alice scans the focal plane and makes a detection, she remotely measures Bob's corresponding photon's momentum and erases its path information, thereby (non-selectively) leaving it as a mixture of plane waves incident on the direction filter. However only the fraction that makes up the pure state comprising the horizontal modes passes through the filter. Diffracting through the double-slit diaphragm, it produces a Young's double slit interference pattern on Bob's screen. Those plane waves coincident with Alice's detecting her photon away from focus f are filtered out and do not reach Bob's double slit assembly. It follows that an interference pattern will emerge in Bob's *singles counts*, coinciding with Alice's detection at f or close to f . Thus Alice can remotely prepare inequivalent ensembles of idlers, depending on whether or not she measures momentum on her photon. In principle, this constitutes a superluminal signal.

Quantitatively, the only difference between the Innsbruck and the proposed experiment is that Eq. (14) is replaced by an expression containing only horizontal modes. As an idealization (to be relaxed below), assuming perfect filtering and low spreading of the wavepacket at the aperture, we have:

$$E_z^{(+)} = e^{ikr_D} \left(e^{ikr_2} \hat{b}_2 + e^{ikr_5} \hat{b}_5 \right), \quad (20)$$

where r_D now represents the distance from the EPR source to the upper/lower slit on Bob's double slit diaphragm (the length of the segment $\overline{qou\bar{x}}$ or \overline{povy}); r_2 (resp., r_5) is the distance from the upper (resp., lower) slit to z . *Detection of a signal photon at or near f is the only possible event on the focal plane such that Bob detects the twin photon at all.* Focal plane detections sufficiently distant from f will project the idler into non-horizontal modes that will be filtered out before reaching Bob's double-slit assembly. Therefore, the interference pattern Eq. (18a) is in fact the only one seen in Bob's singles counts. We denote by $R^F(z)$, this pattern, which Bob obtains conditioned on Alice measuring in the focal plane. By contrast, in the Innsbruck experiment Bob in his singles counts sees a statistical mixture of the patterns (18a), (18b), (18c), etc., corresponding to *all* points on Alice's focal plane illuminated by the signal beam.

When Alice measures in the imaging plane, as in the Innsbruck experiment Bob finds no interference pattern in his singles counts. Setting $\alpha = l, m$ in Eq. (13), and substituting Eqs. (12), (20) and (15) in Eq. (13), we find the coincidence counting rate for detections by Alice and Bob to be

$$R_\alpha(z) \propto \epsilon^2, \quad (\alpha = l, m), \quad (21)$$

which is a uniform pattern (apart from an envelope due to single slit diffraction, which we ignore for the sake of simplicity). It follows that Bob's observed pattern in the singles counts conditioned on Alice measuring in the imaging plane, $R^I(z)$, is also the same, i.e., $R^I(z) \propto \epsilon^2$.

Our main result is the difference between the patterns $R^I(z)$ and $R^F(z)$, which implies that Alice can signal Bob one bit of information across the spacelike interval connecting their measurement events, by choosing to measure her photon in the focal plane or not to measure. In practice, Bob would need to include additional detectors to sample or scan the z -plane fast enough. This procedure can potentially form the basis for a superluminal quantum telegraph, bringing into sharp focus the tension between quantum nonlocality and special relativity.

Considering the far-reaching implications of a positive result to the experiment, we may pause to consider the following: whether our analysis of so far can be correct, and—under the possibility (however limited) that it is—how such a signal may ever arise, in view of the no-signaling theorem. It may be easy to dismiss a proof of putative superluminal communication as 'not even wrong', yet less easy to spot where the purported proof fails and to provide a mechanism for thwarting the signaling. For one, the prediction of the nonlocal signaling is based on a model that departs only slightly from our quantum optical model of Section III A 1, which explains the original Innsbruck experiment quite well. There have been various attempts at proving that quantum nonlocality somehow contravenes special relativity. The author has read some of their accounts, and it was not difficult to spot a hidden erroneous assumption that led to the alleged conflict with relativity. Armed with this lesson, the present claim will be different in the following three ways:

- *We discuss in the following Section various possible objections to our claim, and demonstrate why each of them fails.* Perhaps they do not cover some erroneous but subtle assumption, but even so, our present exercise could still be instructive in yielding new theoretical insights. For example, a proposal for superluminal communication based on light amplification was eventually understood to fail because it violates the no-cloning theorem, a principle that had not been discovered at the time of the proposal was made (cf. [46]).

- *We single out, in the following Section, the key assumption responsible for the superluminality* (that Alice's momentum measurement implements a polynomial superluminal gate). This singling out of the non-standard element at play makes it easier for the reader to judge whether the proposal is wrong, not even wrong, or— as we believe is the case— worth testing experimentally.
- *We have furnished computation- and information-theoretic grounds for why superluminal gates could be possible*, according to which no-signaling could be a nearly-universal-but-not-quite side effect of the computation theoretic properties of physical reality; elsewhere [47], we show how the relativity principle could be a consequence of conservation of information.

In the last Section, we clarify how non-complete measurements, if experimentally validated, could possibly fit in with known physics. There we will argue that they arise owing to the potential fact that practically measurable quantities resulting from quantum field theory are not described by hermitian operators, at variance with a key axiom of orthodox quantum theory [45].

B. The question of existence and origin of the signaling

In the Section, we will consider a number of possible objections to our main result, and demonstrate quantitatively why each of them fails.

a. Spreading at the direction filter. It can be shown that the effect of spreading at the direction filter only lowers—but does not eliminate—the distinguishability between the two kinds of pattern that Bob receives. For illustration, suppose $\delta = 10\lambda$, and as a result, nearly only horizontal modes r_2 and r_5 are selected, but the diffractive spreading at the filter is strong, assumed to be given by $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ in the space spanned by modes 2 and 5, where θ is determined by the geometry of the filter. In place of Eqs. (14) and (16), we now have:

$$E_z^{(+)} = e^{ikr_D} \left(e^{ikr_2} (\cos\theta\hat{b}_2 + \sin\theta\hat{b}_5) + e^{ikr_5} (\cos\theta\hat{b}_5 - \sin\theta\hat{b}_2) \right). \\ R'_\alpha(z) \propto \epsilon^2 [1 \pm \sin(2\theta) \cos(k \cdot [r_2 - r_5])] , \quad (\text{with } \pm \text{ according as } \alpha = l, m). \quad (22)$$

The pattern found by Bob in his singles counts is $R'_l(z) + R'_m(z) \propto \epsilon^2$, which is a constant pattern (ignoring the finite width of the slits), just as when the spreading had been ignored. On the other hand, in place of Eq. (18), we now obtain

$$R'_f(z) \propto \epsilon^2 [1 + \cos(2\theta) \cos(k \cdot [r_2 - r_5])] , \quad (23)$$

Except in the case $\theta = \pi/4$, which is highly unlikely, and in any case, can be precluded by altering δ or G , the two patterns are in principle distinguishable.

b. Alice's focal plane measurement implements a constant gate in the subspace of interest The state (12) is now represented in a simple way as the unnormalized state $|\Psi^{(1)}\rangle = \frac{\epsilon}{\sqrt{6}} \sum_{j=1}^6 |j, j\rangle$, where for simplicity the vacuum state, which does not contribute to the entanglement related effects, is omitted, and it is assumed that each mode contains at most one pair of entangled photons (i.e., no higher excitations of the light field). Further because of the direction filter, it suffices to restrict our attention to the state

$$|\psi^{(2)}\rangle \propto \frac{1}{\sqrt{2}} (|2, 2\rangle + |5, 5\rangle), \quad (24)$$

the projection of $|\Psi^{(1)}\rangle$ onto $\mathcal{H}_2 \otimes \mathcal{H}_2$, where \mathcal{H}_2 is the subspace spanned by $\{|2\rangle, |5\rangle\}$. Under these assumptions, Alice's position measurement in this subspace, represented by the operators \hat{a}_2 and \hat{a}_5 , can be written as the Kraus operators $\hat{a}_2 \equiv |0\rangle\langle 2|$ and $\hat{a}_5 \equiv |0\rangle\langle 5|$. Within \mathcal{H}_2 these operators form a complete set since $\hat{a}_2^\dagger \hat{a}_2 + \hat{a}_5^\dagger \hat{a}_5 = |2\rangle\langle 2| + |5\rangle\langle 5| = \mathbb{I}_2$. Thus, Alice's measurement on $|\Psi^{(2)}\rangle$ in the position basis does not nonlocally affect Bob's reduced density operator, which is proportional to $\mathbb{I}_2/2$.

On the other hand, if Alice measures momentum, her measurement is represented by the field operator $E_f^{(+)}$ in Eq. (17). We have in the above notation

$$E_f^{(+)} \propto \hat{a}_2 + \hat{a}_5 \equiv |0\rangle(\langle 2| + \langle 5|). \quad (25)$$

This is just the polynomial superluminal gate Q in Eq. (10), with the output basis given by $\{|0\rangle, |0^\perp\rangle\}$, where $|0^\perp\rangle$ is any basis element orthogonal to the vacuum state.

We note that the operator $E_f^{(+)} \propto \hat{a}_2 - \hat{a}_5 \equiv |0\rangle\langle 2| - \langle 5|$, that would complete $E_f^{(-)}$ in that $E_f^{(-)}E_f^{(+)} + E_f^{(+)}E_f^{(-)} = \mathbb{I}$ in the space $\text{span}(|2\rangle, |5\rangle)$. However, $E_f^{(+)}$ is necessarily non-physical in the given geometry since modes 2 and 5 meet only at f , where the electric field operator is indeed $\propto \hat{a}_2 - \hat{a}_5$.

We further note that, inspite of the non-completeness of $E_f^{(+)}$, the structure of $|\psi^{(2)}\rangle$ in Eq. (24) guarantees that $E_f^{(+)}|\psi^{(2)}\rangle$ is by default normalized, and hence poses no problem with respect to probability conservation.

By contrast, Bob's measurement is complete (which rules out a Bob-to-Alice superluminal signaling). Each element of Bob's screen z -basis is a possible outcome, described by the annihilation operator approximately of the form $\hat{E}_z^{(-)} \propto \hat{a}_2 + e^{i\gamma}\hat{a}_5$, where $\gamma = \gamma(k, z)$ is the phase difference between the paths 2 and 5 from the slits to a point z on Bob's screen. This represents a POVM of the form $\hat{E}_z^{(-)}\hat{E}_z^{(+)} = (|2\rangle + e^{-i\gamma}|5\rangle)(\langle 2| + e^{i\gamma}\langle 5|)$. Even though $\hat{E}_z^{(+)}$ has the same form as Alice's operator $\hat{E}_f^{(+)}$ —as a Kraus operator describing the absorption of two interfering modes at a point z —, yet, when integrated over his whole ‘position basis’, Bob's measurement is seen to form a complete set, for, as it can be shown, $\int_{z=-\infty}^{+\infty} \hat{E}_z^{(-)}\hat{E}_z^{(+)}dz = |2\rangle\langle 2| + |5\rangle\langle 5|$. In the case of Alice's momentum measurement, because the detection happens at a path singularity, a similar elimination of cross-terms via integration is not possible, whence the non-completeness. It is indeed somewhat intriguing how geometry plays a fundamental role in determining the completeness status of a measurement. This has to do with the fact that the direct detection of a photon is practically a determination of *position* distribution. For example, even in remotely measuring the idler's momentum, Alice measures her photon's position at the focal plane. We will return again to this issue in the final Section.

c. Role of the direction filter. A simple model of the action of the perfect direction filter is

$$D \equiv \sum_{j=2,5} |j\rangle\langle j| + \sum_{j \neq 2,5} |-j\rangle\langle j| \quad (26)$$

acting locally on the second register of the state of $|\Psi^{(1)}\rangle$. Here $|-j\rangle$ can be thought of as a state orthogonal to all $|j\rangle$'s and other $|-j\rangle$'s, that removes the photon from the experiment, for example, by reflecting it out or by absorption at the filter. It suffices for our purpose to note that D can be described as a local, standard (linear, unitary and hence complete) operation. Since the structure of QM guarantees that such an operation cannot lead to nonlocal signaling, the conclusion is that the superluminal signal, if it exists, must remain *even if the direction filter is absent*.

We will employ the notation $|j+k+m\rangle \equiv (1/\sqrt{3})(|j\rangle + |k\rangle + |m\rangle)$. To see that the nonlocal signaling is implicit in the state modified by Alice's actions even without the application of the filter, we note the following: if Alice measures ‘momentum’ on the state $|\Psi^{(1)}\rangle$ and detects a signal photon at f , she projects the corresponding idler into the state $|2+5\rangle$. Similarly, her detection of a photon at f'' projects the idler into the state $|3+6\rangle$, and her detection at f' , projects the idler into the state $|1+4\rangle$. Therefore, in the absence of the direction filter, Alice's remote measurement of the idler's momentum leaves the idler in a (assumed uniform for simplicity) mixture given by

$$\rho_P \propto |2+5\rangle\langle 2+5| + |1+4\rangle\langle 1+4| + |3+6\rangle\langle 3+6|. \quad (27)$$

Her momentum measurement is non-complete, since the summation over the corresponding projectors (r.h.s of Eq. (27)) is not the identity operation \mathbb{I}_6 pertaining to the Hilbert space spanned by six modes $|j\rangle$ ($j = 1, \dots, 6$).

On the other hand, if Alice remotely measures the idler's position, she leaves the idler in the mixture

$$\rho_Q \propto |1+2+3\rangle\langle 1+2+3| + |4+5+6\rangle\langle 4+5+6|. \quad (28)$$

Here again, her position measurement is non-complete, reflected in the fact that the summation over the corresponding projectors (r.h.s of Eq. (28)) is not \mathbb{I}_6 [48].

Since $\rho_P \neq \rho_Q$, we are led to conclude that the violation of no-signaling *is already implicit in the Innsbruck experiment*. Yet, since Bob measures in the z -basis rather than the ‘mode’ basis, in the absence of a direction filter—as is the case in the Innsbruck experiment—, Bob's screen will not register any signal, for the following reason. In case of Alice's focal plane measurement, the integrated diffraction-interference pattern corresponding to different outcomes will wash out any observable interference pattern. On the other hand, in the case of Alice's imaging plane measurement, each of Bob's detections comes from the photon's incoherent passage through one or the other slit, and hence—again—no interference pattern is produced on his screen. Thus, measurement at Bob's screen plane z without a direction filter will render ρ_P effectively indistinguishable from ρ_Q . The role played by the direction filter is to prevent modal averaging in case of Alice's momentum measurement, by selecting one set of modes. The filter does not create, but only exposes, a superluminal effect that otherwise remains hidden.

d. Complementarity of single- and two-particle correlations. It is well known that path information (or particle nature) and interference (or wave nature) are mutually exclusive or complementary. In the two-photon case, this takes the form of mutual incompatibility of single- and two-particle interference [49, 50], because entanglement can be used

to monitor path information of the twin particle, and is thus equivalent to ‘particle nature’. One may thus consider single- and two-particle correlations as being related by a kind of complementarity relation that parallels wave- and particle-nature complementarity. A brief exposition of this idea is given in the following paragraph.

For a particle in a double-slit experiment, we restrict our attention to the Hilbert space \mathcal{H} , spanned by the state $|0\rangle$ and $|1\rangle$ corresponding to the upper and lower slit of a double slit experiment. Given density operator ρ , we define coherence C by $C = 2|\rho_{01}| = 2|\rho_{10}|$, a measure of cross-terms in the computational basis not vanishing. The particle is initially assumed to be in the state $|\psi_a\rangle$, and a “monitor”, initially in the state $|0\rangle$, interacting with each other by means of an interaction U , parametrized by variable θ that determines the entangling strength of U . It is convenient to choose $U = \cos\theta I \otimes I + i\sin\theta \text{CNOT}$, where CNOT is the operation $I \otimes |0\rangle\langle 0| + X \otimes |1\rangle\langle 1|$, where X is the Pauli X operator. Under the action of U , the system particle goes to the state $\rho = \text{Tr}_m[U(|\psi_a\rangle|0\rangle\langle\psi_a|\langle 0|)U^\dagger] = \frac{I}{2} + \frac{1}{2}[(\cos\theta + i\sin\theta)\cos\theta|0\rangle\langle 1| + \text{c.c.}]$, where $\text{Tr}_m[\dots]$ indicates taking trace over the monitor. Applying the above formula for coherence to ρ , we calculate that coherence $C = \cos\theta$. We let λ_\pm denote the eigenvalues of ρ . Quantifying the degree of entanglement by concurrence [56], we have $E \equiv 2\sqrt{\lambda_- \lambda_+} = \sin\theta$. We thus obtain a trade-off between coherence and entanglement given by $C^2 + E^2 = 1$, a manifestation of the complementarity between single-particle and two-particle interference.

In the context of the proposed experiment, this could raise the following purported objection to our proposed signaling scheme: as the experiment happens in the near-field regime, where two-particle correlations are strong, one would expect that Bob should not find an interference pattern in his singles counts. Yet, contrary to this expectation, Eq. (18) implies that such an interference pattern does appear. The reason is that in the focal plane measurement, Alice is able to erase her path information in the subspace \mathcal{H}_2 , but, by virtue of the associated non-completeness, she does so in *only one* way, viz. via the non-complete operation $E_f^{(+)}$ associated with her measurement. If her measurement were *complete*, she would erase path information in more than one way, and the corresponding conditional single-particle interference patterns would mutually cancel each other in the singles count. This is clarified in the following Section.

e. Polarization and ‘interferometric quantum computing’. Q -like gates describe the situation where two converging modes at the path singularity have the *same* polarization. The quantum optics formalism implies that if the polarizations of the two incoming modes are not parallel when interfering, then the polarization states add vectorially (that is, superpose), with amplitudes being added componentwise along each polarization/dimension, and the resulting probability being the squared magnitude of this vector sum. One can define a corresponding more general constant gate (a tensor sum of constant gates over the internal dimensions), and a correspondingly potentially larger BQP_c . It can be shown that Theorem 1 still holds. Here we will content ourselves to illustrate it by a simple example.

Suppose we have this ‘interferometric quantum computer’: a 2^n -level atom, whose spin part is prepared initially in the state $|a\rangle \equiv (2^{-n/2})(|1\rangle + |2\rangle + \dots + |2^n-1\rangle + |2^n\rangle)$. The spatial part of the atom’s matter wave is now split into two subwaves by an appropriate beam-splitter, and then refocused onto a path singularity. On the second subwave, before the two subwaves reach the region of spatial overlap, an oracle operation is applied which in a single step inverts the sign of all the kets, except the ‘marked’ state $|2^n\rangle$, yielding $|b\rangle \equiv (2^{-n/2})(-|1\rangle - |2\rangle - \dots - |2^n-1\rangle + |2^n\rangle)$. According to the above prescription, the output at the path singularity should be $|a\rangle + |b\rangle = 2|2^n\rangle/2^{n/2}$, i.e., a particle is detected with exponentially low probability $\| |a\rangle + |b\rangle \|^2 = 4 \cdot 2^{-n}$, and detection leaves the particle in the state $|2^n\rangle$. The oracle together with detection at the path singularity is equivalent to the non-complete operation $\bigoplus_{j=1}^{2^n-1} Q_2^{(j)}(\pi) \oplus Q_2^{(2^n)}(0)$.

If the marked state is designated to be a possible solution to a SAT problem, the measurement would have to be repeated an exponentially large number of times, or performed once on an exponentially large number of atoms, to detect a possible ‘yes’ outcome. Either way, the physical situation is compatible with the WNHE assumption, but not with no-signaling. (We observe that augmenting the detection with a renormalization following vector addition would in fact implement the post-selection gate.)

Finally, let us clarify the sense in which non-complete operations like Q of potential physical interest may *effectively* conform to probability conservation. In the Modified Innsbruck experiment, Alice’s application of Q conforms *exactly* to probability conservation, because the state $|\psi^{(2)}\rangle$ in Eq. (24) has a Schmidt form, with Q defined in Bob’s Schmidt basis. However, this is not the general situation. In such cases, one seems to find that the spreading of the wavefunction produces a pattern of bright and dark interferometric fringes at and around the path singularity such that, even though locally there is an excess or deficit over the average probability density, still there is an overall probability conservation across the fringes. This is somewhat comparable to the situation with Bob’s POVM $\hat{E}_z^{(-)} \hat{E}_z^{(+)}$, which, even though locally a Q -like operation, still yields identity when integrated over z . This conservation mechanism is not applicable to the Modified Innsbruck experiment, which is performed in the near-field limit, where spreading is minimal and two-particle correlations are strong. However, as noted above, probability conservation is inherently exact for the situation in the experiment, and the mechanism need not be invoked.

As an illustration of the mechanism, let the angle at which the two interfering beams of the ‘interferometric quantum computer’ converge towards a spatial overlap region be θ . The fringes are given by a stationary pattern with spatial

frequency $k' = k \sin \theta \approx k\theta = k(S/d)$, where S is the spatial separation between two optical elements (say, mirrors) that are, respectively, reflecting the beams along the two interferometric arms towards q , and d the distance from the central point between these mirrors to the center of region q . The width of each fringe is about $2\pi/k' = \lambda(d/S)$. Now the initial beam width must be of the order of several wavelengths, and the diffractive spread rate of each beam at least λ/S , so that beam width $> \lambda d/S$. Thus, the spreading of (quantum) waves guarantees that there will always be compensatory fringes, and hence overall conservation of probability, even though locally the dark and bright bands contain less or more than the average probability density.

Applied to the above atom interferometer example, the state vector at the interference screen will have the form $\kappa(\theta)(|a\rangle + e^{i\theta}|b\rangle)$ with θ running from $-\infty$ to $+\infty$, where $\kappa(\theta)$ is a narrow Gaussian-like function centered at $\theta = 0$. When $\theta = 0, 2\pi, 4\pi, \dots$, one obtains dark fringes with the ‘solution’ $|2^n\rangle$ at exponentially low intensity, as noted above. When $\theta = \pi, 3\pi, 5\pi, \dots$, one obtains bright fringes of nearly maximal intensity, diminished by only an exponentially small amount, corresponding, again, to the ‘solution’. Thus the interference pattern is a band of bright and dark fringes at spatial frequency k' with the bright ones very slightly dimmer than if $|a\rangle$ and $|b\rangle$ had the same polarization, and the dark ones very slightly brighter.

IV. DISCUSSIONS AND CONCLUSIONS

Considering the far-reaching implications of a positive result to our proposed experiment, even though we have ruled out in Section IIIB all the (as far as we know) obvious objections, we have to remain open to the possibility that there may be a subtle error, possibly a hidden unwarranted assumption, somewhere in our analysis. In the surprising event that the proposed experiment yields a positive outcome, no-signaling would no longer be a universal condition, and the issue of ‘speed of quantum information’ [51] would assume practical significance. It would also bolster the case for believing that the WNHE assumption is a basic principle of quantum physics, and that considerations of intractability, and by extension uncomputability, can serve as an informal guide to basic physics.

Physical space would be regarded as a type of information, and physical dynamics a kind of computation, with physical separation being not genuine obstacle to rapid communication in the way it would be when seen from the perspective of causality in conventional physics. On the other hand, the barrier between polynomial-time and hard problems would be real, and the physical existence of superluminal signals would thus not be as surprising as that of exponential gates. Interestingly, polynomial superluminal operations exist even in classical computation theory. The Random Access Machine (RAM) model [52], a standard model in computer science wherein memory access takes exactly one time-step irrespective of the physical location of the memory element, illustrates this idea. RAMs are known to be polynomially equivalent to Turing machines.

Even granting that the noncomplete gate Q' turns out to be physically valid and realizable, this brings us to another important issue: how would non-completeness fit in with the known mathematical structure of the quantum properties of particles and fields? We venture that the answer has to do with the nature of and relationship between observables in QM on the one hand, and those in quantum optics, and more generally, in quantum field theory (QFT), on the other hand.

It is frequently claimed that QFT is just the standard rules of first quantization applied to classical fields, but this position can be criticized [45, 53, 54]. For example, the relativistic effects of the integer-spin QFT imply that the wavefunctions describing a fixed number of particles do not admit the usual probabilistic interpretation [54]. Again, fermionic fields do not really have a classical counterpart and do not represent quantum observables [45].

In practice, measurable properties resulting from a QFT are properties of particles—of photons in quantum optics. Particulate properties such as number, described by the number operator constructed from fields, or the momentum operator, which allows the reproduction of single-particle QM in momentum space, do not present a problem. The problem is the *position* variable, which is considered to be a parameter, and not a Hermitian operator, both in QFT and single-particle relativistic QM, and yet relevant experiments measure particle positions. The experiment described in this work involves measurement of the positions of photons, as for example, Alice’s detection of photons at points on the imaging or focal plane, or Bob’s detection at points on the z -plane, respectively. There seems to be no way to derive from QFT the experimentally confirmed Born rule that the nonrelativistic wavefunction $\psi(\mathbf{x}, t)$ determines quantum probabilities $|\psi(\mathbf{x}, t)|^2$ of particle positions. In most practical situations, this is really not a problem. The probabilities in the above experiment were computed according to standard quantum optical rules to determine the correlation functions at various orders [55], which serve as an effective wavefunction of the photon, as seen for example from Eqs. (13). In QFT, particle physics phenomenologists have developed intuitive rules to predict distributions of particle positions from scattering amplitudes in *momentum* space.

Nevertheless, there is a problem in principle, and this leads us to ask whether QFT is a genuine quantum theory [45]. If we accept that properties like position are valid observables in QM, the answer seems to be ‘no’. We see this again in the fact that the effective ‘momentum’ and ‘position’ observables that arise in the above experiment

are not seen to be Hermitian operators of standard QM (cf. note [48]). Further, non-complete operations like $\hat{E}_f^{(+)}$, disallowed in QM, seem to appear in QFT. This suggests that it is QM, and not QFT, that is proved to be strictly non-signaling by the no-signaling theorem.

Since nonrelativistic QM and QFT are presumably not two independent theories describing entirely different objects, but do describe the same particles in many situations, the relationship between observables in the two theories needs to be better understood. Perhaps some quantum mechanical observables are a coarse-graining of QFT ones, having wide but not universal validity. For example, Alice's detection of a photon at a point in the focal plane was quantum mechanically understood to project the state of Bob's photon into a one-dimensional subspace corresponding to a momentum eigenstate. Quantum optically, however, this 'eigenstate' is described as a superposition of a number of parallel, in-phase modes originating from different down-conversion events in the non-linear crystal, producing a coherent plane wave propagating in a particular direction.

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 - [19] That is, 'the universe is not hard enough to *not* be simulable using polynomial resources'. The expression is non-technically related to the statement "The world is not enough" ("*orbis non sufficit*"), the family motto of, as well as a motion picture featuring, a well known Anglo-Scottish secret agent!
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- [33] In complexity theory, **PP** is the class of decision problems for which there exists a polynomial time probabilistic TM such that: if the answer is ‘yes’, it returns ‘yes’ with probability greater than $1/2$, and if the answer is ‘no’, it returns ‘yes’ with probability at most $1/2$.
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